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SIMULTANEOUS ESTIMATION OF THE SPACEWISE AND TIMEWISE VARIATIONS OF MASS AND HEAT TRANSFER COEFFICIENTS IN DRYING

Leonardo F. Saker and Helcio R. B. Orlande
Department of Mechanical Engineering, DEM/PEM - POLI/COPPE
Federal University of Rio de Janeiro, UFRJ
Rio de Janeiro, RJ, Brazil
saker@petrobras.com.br and helcio@serv.com.ufrj.br

Cheng-Hung Huang
Department of Systems & Naval Mechatronic Engineering
National Cheng Kung University
Taiwan
chhuang@mail.ncku.edu.tw

Gligor H. Kanevce
Macedonian Academy of Sciences and Arts
Skopje, Macedonia
kanevce@osi.net.mk

Ljubica P. Kanevce
Faculty of Technical Sciences
St. Kliment Ohridski University
Bitola, Macedonia
kanevce@osi.net.mk

ABSTRACT

In this paper, the conjugate gradient method with adjoint problem is applied for the identification of the heat and mass transfer coefficients at the surface of drying capillary-porous bodies. The unknown functions are supposed to vary in time and along the surface open to the surrounding environment. The effects of temperature and moisture content measurements on the inverse analysis are examined. The inverse problem is solved by considering either the heat or the mass transfer coefficients as unknown, as well as by considering simultaneously both functions as unknown.

NOMENCLATURE

Bi_m	mass transfer coefficient
Bi_q	heat transfer coefficient
C	measured moisture content
Ko	Kossovitch number
Lu	Luikov number
M	measured temperature
Pn	Posnov number
r_a	aspect ratio
S	objective functional
X,Y	dimensionless Cartesian coordinates

Greeks

Df	sensitivity function for moisture content
Dq	sensitivity function for temperature
y^k	conjugation coefficient def. by Eq. (6.b)
a	parameter defined by Eq. (2.m)
b	parameter defined by Eq. (2.n)
b^k	search step size at iteration k
f	dimensionless moisture content
g	parameter defined by Eq. (2.o)
g^k	conjugation coefficient def. by Eq. (6.a)
l	Lagrange multiplier
q	dimensionless temperature
t	dimensionless time

INTRODUCTION

The phenomena of coupled heat and mass transfer in capillary porous media has been drawing the attention of research groups for a long time, because of its importance in several practical applications, such as drying. For the mathematical modeling of such phenomena, Luikov [1] has proposed a model based on a system of coupled diffusion equations, which takes into account the effects of the temperature gradient on the moisture migration.

The computation of temperature and moisture content fields in capillary porous media, from the knowledge of initial and boundary conditions, as well as of the thermophysical properties appearing in the formulation, constitutes a *Direct Problem* of heat and mass transfer [1,2]. Appropriately formulated direct problems are mathematically classified as *well-posed*, that is, their solutions satisfy the requirements of existence, uniqueness and stability with respect to the input data [3-5]. On the other hand, the estimation of boundary conditions in Luikov's formulation, by using temperature and/or moisture content measurements taken in the medium, is an *Inverse Problem* of coupled heat and mass transfer [3-5]. Generally, inverse problems are mathematically classified as *ill-posed* [3-5]. Despite the ill-posed character, the solution of an inverse problem can be obtained through its reformulation in terms of a well-posed problem, such as a minimization problem associated with some kind of regularization (stabilization) technique. Different methods based on such an approach have been successfully used in the past for the estimation of parameters and functions, in linear and non-linear inverse problems [3-5]. Recently, several articles dealing with the solution of inverse problems of coupled heat and mass transfer appeared in the literature [6-18].

In this paper, we examine the solution of inverse problems involving the identification of the heat and mass transfer coefficients at the surface of a capillary-porous body, as a function estimation approach. The unknown quantities are supposed to vary in time and along the surface of the body that is open to the surrounding environment. The conjugate gradient method with adjoint problem [3-5] is used for the identification of the unknown functions. The inverse problems of estimating one single function are examined, as well as the inverse problem of simultaneously estimating both functions. The effects of the use of temperature and moisture content measurements on the inverse analysis are examined, by using simulated measured data with random errors.

PHYSICAL PROBLEM AND MATHEMATICAL FORMULATION

The physical problem involves a two-dimensional capillary porous medium in Cartesian coordinates, initially at uniform temperature and uniform moisture content. The lateral surfaces of the body are impervious to moisture transfer and thermally insulated. The bottom boundary, which is impervious to moisture transfer, is in direct contact with a heater. The top boundary is in contact with the dry surrounding air, thus resulting in a convective boundary condition for both the temperature and the moisture content. The mass and heat transfer coefficients at this boundary may vary along the surface open to the surrounding environment. The linear system of equations proposed by Luikov [1], with associated initial and boundary conditions, for the modeling of such physical problem involving heat and mass transfer in capillary porous media, can be written in dimensionless form as:

$$\frac{\partial q}{\partial t} = a\left(\frac{\partial^2 q}{\partial X^2} + \frac{\partial^2 q}{\partial Y^2}\right) - b\left(\frac{\partial^2 q}{\partial X^2} + \frac{\partial^2 q}{\partial Y^2}\right)$$

in $0 < X < r_a$, $0 < Y < 1$ and $t > 0$ (1.a)

$$\frac{\partial f}{\partial t} = Lu\left(\frac{\partial^2 f}{\partial X^2} + \frac{\partial^2 f}{\partial Y^2}\right) - Lu\,Pn\left(\frac{\partial^2 q}{\partial X^2} + \frac{\partial^2 q}{\partial Y^2}\right)$$

in $0 < X < r_a$, $0 < Y < 1$ and $t > 0$ (1.b)

$$\frac{\partial q}{\partial Y} = -Q \qquad \text{at } Y = 0 \text{ , for } t > 0 \qquad (1.c)$$

$$\frac{\partial f}{\partial Y} = -Pn\,Q \qquad \text{at } Y = 0 \text{ , for } t > 0 \qquad (1.d)$$

$$\frac{\partial q}{\partial Y} = Bi_q(X,t)(1-q) - (1-e)Ko\,Lu\,Bi_m(X,t)(1-f)$$

at $Y = 1$, for $t > 0$ (1.e)

$$\frac{\partial f}{\partial Y} = Pn\frac{\partial q}{\partial Y} + Bi_m(X,t)(1-f)$$

at $Y = 1$, for $t > 0$ (1.f)

$$\frac{\partial q}{\partial X} = \frac{\partial f}{\partial X} = 0 \qquad \text{at } X = 0 \text{ and } X = r_a \text{, for } t > 0$$

(1.g,h)

$$q(X,Y,0) = f(X,Y,0) = 0$$

for $t = 0$, in $0 < X < r_a$, $0 < Y < 1$ (1.i,j)

where the following dimensionless variables were defined:

$$q = \frac{T - T_0}{T_s - T_0} \qquad f = \frac{u_0 - u}{u_0 - u^*} \qquad Q = \frac{qh}{k(T_s - T_0)}$$

$$t = \frac{at}{h^2} \qquad Lu = \frac{a_m}{a} \qquad Pn = d\frac{T_s - T_0}{u_0 - u^*}$$

$$Ko = \frac{r(u_0 - u^*)}{c(T_s - T_0)} \qquad X = \frac{x}{h} \qquad Y = \frac{y}{h} \qquad r_a = \frac{L}{h}$$

$$Bi_q(X,t) = \frac{h_q(X,t)\,h}{k} \qquad Bi_m(X,t) = \frac{h_m(X,t)\,h}{k_m}$$

$$a = 1 + e\,Ko\,Lu\,Pn \qquad b = e\,Ko\,Lu$$

$$g = 1 - (1-e)Ko\,Lu\,Pn \qquad (2.a\text{-}o)$$

The properties of the porous medium appearing above include the thermal diffusivity (a), the moisture diffusivity (a_m), the thermal conductivity (k), the moisture conductivity (k_m)

and the specific heat (*c*). Other physical quantities appearing in the dimensionless groups of Eqs. (2) are the heat transfer coefficient (h_q), the mass transfer coefficient (h_m), the thickness of porous medium (*h*), the length of the porous medium (*L*), the prescribed heat flux (*q*), the latent heat of evaporation of water (*r*), the temperature of the surrounding air (T_s), the uniform initial temperature in the medium (T_0), the moisture content of the surrounding air (u^*), the uniform initial moisture content in the medium (u_0), the thermogradient coefficient (*d*) and the phase conversion factor (*e*). *Lu*, *Pn* and *Ko* denote the Luikov, Posnov and Kossovitch numbers, respectively [1].

Problem (1) is referred to as a *Direct Problem* when initial and boundary conditions, as well as all parameters appearing in the formulation, are known. The objective of the direct problem is to determine the dimensionless temperature and moisture content fields, $q(X,Y,t)$ and $f(X,Y,t)$, respectively, in the capillary porous media.

INVERSE PROBLEM

For the *inverse problem* of interest here, the functions $Bi_q(X,t)$ and $Bi_m(X,t)$ are regarded as unknown quantities. For the estimation of such functions, we consider available the transient temperature measurements $M_i(t)$ taken at the locations (X_i,Y_i) $i=1,\ldots,I$, as well as the moisture content measurements $C_n(t)$ taken at the locations (X_n^*,Y_n^*), $n=1,\ldots,N$. We note that the measurements may contain random errors, but all the other quantities appearing in the formulation of the direct problem are supposed to be exactly known.

Inverse problems are ill-posed [3-5]. Several methods of solution of inverse problems, such as the one used here, involve their reformulation in terms of well-posed minimization problems. We consider our objective functional in the form:

$$S[Bi_m(X,t),Bi_q(X,t)] = \int_{t=0}^{t_f}\left\{\sum_{i=1}^{I}[q(X_i,Y_i,t;Bi_m,Bi_q)-M_i(t)]^2 w_q\right\}dt + \int_{t=0}^{t_f}\left\{\sum_{n=1}^{N}[f(X_n^*,Y_n^*,t;Bi_m,Bi_q)-C_n(t)]^2 w_f\right\}dt \tag{3}$$

where q and f are the estimated temperature and moisture content, respectively, which are obtained from the solution of the direct problem with estimates for the unknown functions. In equation (3) w_q and w_f are weights for the temperature and moisture content measurements, respectively.

For the minimization of such objective functional, we use here the conjugate gradient method with adjoint problem, as described next.

CONJUGATE GRADIENT METHOD

The iterative procedure of the conjugate gradient method is given by:

$$Bi^{k+1}(X,t) = Bi^k(X,t) + b^k d^k(X,t) \tag{4}$$

where the superscript *k* denotes the number of iterations, b^k is the search step size, $d^k(X,t)$ is the direction of descent and $Bi(X,t)$ may represent $Bi_q(X,t)$ and $Bi_m(X,t)$.

The direction of descent $d^k(X,t)$ is a conjugation of the gradient direction with previous directions of descent. It is given in the following general form:

$$d^k(X,t) = -\nabla S[Bi^k(X,t)] + g^k d^{k-1}(X,t) + y^k d^q(X,t) \tag{5}$$

where g^k and y^k are conjugation coefficients. The superscript q in equation (5) denotes the iteration number where a *restarting strategy* is applied to the iterative procedure of the conjugate gradient method.

Different versions of the conjugate gradient method can be found in the literature depending on the form used for the computation of the direction of descent given by equation (5) [3-5, 19,20]. In this paper, the so-called Powell-Beale's version is used due to its superior robustness for the solution of non-linear inverse problems [20].

Powell [19] suggested the following expressions for the conjugation coefficients, which gives the so-called *Powell-Beale's* version of the conjugate gradient method [19,20]:

$$g^k = \frac{\int_{t=0}^{t_f}\int_{X=0}^{r_a}\{\nabla S[Bi^k]-\nabla S[Bi^{k-1}]\}\nabla S[Bi^k]\,dX\,dt}{\int_{t=0}^{t_f}\int_{X=0}^{r_a}\{\nabla S[Bi^k]-\nabla S[Bi^{k-1}]\}\,d^{k-1}(X,t)\,dX\,dt}$$

with $g^k = 0$ for $k = 0$ (6.a)

$$y^k = \frac{\int_{t=0}^{t_f}\int_{X=0}^{r_a}\{\nabla S[Bi^{q+1}]-\nabla S[Bi^q]\}\nabla S[Bi^k]\,dX\,dt}{\int_{t=0}^{t_f}\int_{X=0}^{r_a}\{\nabla S[Bi^{q+1}]-\nabla S[Bi^q]\}\,d^q(X,t)\,dX\,dt}$$

with $y^k = 0$ for $k = 0$ (6.b)

In accordance with Powell [19], the application of the conjugate gradient method with the conjugation coefficients given by equations (6) requires restarting when gradients at successive iterations tend to be non-orthogonal (which is a measure of the local non-linearity of the problem) and when the direction of descent is not sufficiently downhill. Restarting is performed by making $y^k = 0$ in equation (5). For more details on the restarting strategy see references [18-20].

For the numerical implementation of the iterative procedure of the conjugate gradient method, auxiliary problems are required, namely the sensitivity problems and the adjoint problem, as described next.

SENSITIVITY PROBLEMS AND SEARCH STEP SIZE

The sensitivity problem is used to determine the variation in temperature and in moisture content due to changes in the unknown quantity. Since the present work deals with the estimation of two unknown functions, two sensitivity problems are required in the analysis. They are derived by considering perturbations in the heat and mass transfer coefficients, each at a time, as described next.

Let us consider that the temperature $q(X,Y,t)$ and the moisture content $f(X,Y,t)$ undergo variations $\Delta q_1(X,Y,t)$ and $\Delta f_1(X,Y,t)$, respectively, when the mass transfer coefficient $Bi_m(X,t)$ is perturbed by $\Delta Bi_m(X,t)$. By substituting in the direct problem (1) $q(X,Y,t)$ by $[q(X,Y,t) + \Delta q_1(X,Y,t)]$, $f(X,Y,t)$ by $[f(X,Y,t)+ \Delta f_1(X,Y,t)]$ and $Bi_m(X,t)$ by $[Bi_m(X,t)+DBi_m(X,t)]$, and then subtracting from the resulting problem the original direct problem, we obtain the following sensitivity problem for the sensitivity functions $\Delta q_1(X,Y,t)$ and $\Delta f_1(X,Y,t)$:

$$\frac{\partial \Delta q_1}{\partial t} = a\left(\frac{\partial^2 \Delta q_1}{\partial X^2} + \frac{\partial^2 \Delta q_1}{\partial Y^2}\right) - b\left(\frac{\partial^2 \Delta q_1}{\partial X^2} + \frac{\partial^2 \Delta q_1}{\partial Y^2}\right)$$

in $0 < X < r_a$, $0 < Y < 1$ and $t > 0$ (7.a)

$$\frac{\partial \Delta f_1}{\partial t} = Lu\left(\frac{\partial^2 \Delta f_1}{\partial X^2} + \frac{\partial^2 \Delta f_1}{\partial Y^2}\right) - Lu\,Pn\left(\frac{\partial^2 \Delta q_1}{\partial X^2} + \frac{\partial^2 \Delta q_1}{\partial Y^2}\right)$$

in $0 < X < r_a$, $0 < Y < 1$ and $t > 0$ (7.b)

$$\frac{\partial \Delta q_1}{\partial Y} = \frac{\partial \Delta f_1}{\partial Y} = 0$$ at $Y = 0$, for $t > 0$ (7.c.d)

$$\frac{\partial \Delta q_1}{\partial Y} = -Bi_q(X,t)\Delta q_1 + (1-e)Ko\,Lu\,\Delta Bi_m(X,t)[f-1] + (1-e)Ko\,Lu\,Bi_m(X,t)\,\Delta f_1$$

at $Y = 1$, for $t > 0$ (7.e)

$$\frac{\partial \Delta f_1}{\partial Y} = \{\Delta Bi_m(X,t) - Bi_m(X,t)\Delta f_1 - fBi_m(X,t)\}g - Bi_q(X,t)\,Pn\,\Delta q_1$$

at $Y = 1$, for $t > 0$ (7.f)

$$\frac{\partial \Delta q_1}{\partial X} = \frac{\partial \Delta f_1}{\partial X} = 0$$

at $X = 0$ and $X = r_a$, for $t > 0$ (7.g,h)

$$\Delta q_1(X,Y,0) = \Delta f_1(X,Y,0) = 0$$

for $t = 0$, in $0 < X < r_a$, $0 < Y < 1$ (7.i,j)

Similarly, the sensitivity problem for the sensitivity functions $\Delta q_2(X,Y,t)$ and $\Delta f_2(X,Y,t)$, resultant from a perturbation $DBi_q(X,t)$ in $Bi_q(X,t)$, can be obtained as:

$$\frac{\partial \Delta q_2}{\partial t} = a\left(\frac{\partial^2 \Delta q_2}{\partial X^2} + \frac{\partial^2 \Delta q_2}{\partial Y^2}\right) - b\left(\frac{\partial^2 \Delta q_2}{\partial X^2} + \frac{\partial^2 \Delta q_2}{\partial Y^2}\right)$$

in $0 < X < r_a$, $0 < Y < 1$ and $t > 0$ (8.a)

$$\frac{\partial \Delta f_2}{\partial t} = Lu\left(\frac{\partial^2 \Delta f_2}{\partial X^2} + \frac{\partial^2 \Delta f_2}{\partial Y^2}\right) - Lu\,Pn\left(\frac{\partial^2 \Delta q_2}{\partial X^2} + \frac{\partial^2 \Delta q_2}{\partial Y^2}\right)$$

in $0 < X < r_a$, $0 < Y < 1$ and $t > 0$ (8.b)

$$\frac{\partial \Delta q_2}{\partial Y} = \frac{\partial \Delta f_2}{\partial Y} = 0$$ at $Y = 0$, for $t > 0$ (8.c.d)

$$\frac{\partial \Delta q_2}{\partial Y} = \Delta Bi_q(X,t) - [Bi_q(X,t)\Delta q_2 + q\Delta Bi_q(X,t)] + (1-e)Ko\,Lu\,Bi_m(X,t)\Delta f_2$$

at $Y = 1$, for $t > 0$ (8.e)

$$\frac{\partial \Delta f_2}{\partial Y} = -Bi_m(X,t)\Delta f_2\,g + \Delta Bi_q(X,t)\,Pn - [Bi_q(X,t)\Delta q_2 + q\Delta Bi_q(X,t)]Pn$$

at $Y = 1$, for $t > 0$ (8.f)

$$\frac{\partial \Delta q_2}{\partial X} = \frac{\partial \Delta f_2}{\partial X} = 0$$

at $X = 0$ and $X = r_a$, for $t > 0$ (8.g,h)

$$\Delta q_2(X,Y,0) = \Delta f_2(X,Y,0) = 0$$

for $t = 0$, in $0 < X < r_a$, $0 < Y < 1$ (8.i,j)

Linearized expressions can be obtained for the search step-sizes for the iterative procedures for the estimation of $Bi_q(X,t)$ and $Bi_m(X,t)$, by minimizing the objective functional at each iteration with respect to these quantities. We omit details of such derivations, but they can be readily found in Ref. [5].

ADJOINT PROBLEM AND GRADIENT EQUATIONS

The adjoint problem is derived by multiplying the governing equations of the direct problem by Lagrange multipliers, integrating in the spatial and time domains that they are valid and then adding the resultant equation to the original functional (3). The directional derivative of the functional in the direction of the perturbation of each of the unknown functions is then obtained and the resultant expression, after some lengthy but straightforward manipulations, is allowed to go to zero. The same adjoint problem is obtained for perturbations in $Bi_q(X,t)$ and $Bi_m(X,t)$. The adjoint problem, for the computation of the Lagrange Multipliers $\lambda_1(X,Y,t)$ and $\lambda_2(X,Y,t)$, is given by:

$$\frac{\partial l_1}{\partial t} = a\left(\frac{\partial^2 l_1}{\partial X^2} + \frac{\partial^2 l_1}{\partial Y^2}\right) + Lu\ Pn\left(\frac{\partial^2 l_2}{\partial X^2} + \frac{\partial^2 l_2}{\partial Y^2}\right) - \sum_{i=1}^{I} 2\left(q_i - M_i\right) d\,(X - X_i)\, d\,(Y - Y_i)\, w_q$$

in $0 < X < r_a$, $0 < Y < 1$ and $t > 0$ (9.a)

$$\frac{\partial l_2}{\partial t} = Lu\left(\frac{\partial^2 l_1}{\partial X^2} + \frac{\partial^2 l_1}{\partial Y^2}\right) + b\left(\frac{\partial^2 l_2}{\partial X^2} + \frac{\partial^2 l_2}{\partial Y^2}\right) - \sum_{n=1}^{N} 2\left(f_n - C_n\right) d\,(X - X_n^*)\, d\,(Y - Y_n^*)\, w_f$$

in $0 < X < r_a$, $0 < Y < 1$ and $t > 0$ (9.b)

$$\frac{\partial l_1}{\partial Y} = \frac{\partial l_2}{\partial Y} = 0 \quad \text{at } Y = 0 \text{ , for } t > 0 \qquad (9.c.d)$$

$$\frac{\partial l_1}{\partial Y} = -\, l_1\, Bi_q\,(X\,,t\,) - \frac{Lu\ \ l_2\ Bi_q\,(X\,,t\,)\ Pn}{a}$$

at $Y = 1$, for $t > 0$ (9.e)

$$\frac{\partial l_2}{\partial Y} = -a\ \ l_1\,(1-e)\,Ko\,Bi_m\,(X\,,t\,) - l_2\,Bi_m\,(X\,,t\,)g$$

at $Y = 1$, for $t > 0$ (9.f)

$$\frac{\partial l_1}{\partial Y} = \frac{\partial l_2}{\partial Y} = 0$$

at $X = 0$ and $X = r_a$, for $t > 0$ (9.g,h)

$$l_1(X,Y,t_f) = l_2(X,Y,t_f) = 0$$

for $t = t_f$, in $0 < X < r_a$, $0 < Y < 1$ (9.i,j)

With the limiting process used to obtain the adjoint problem (9), we can also identify the following expressions for the gradient directions, where it was taken into account the hypotheses that $Bi_q(X,t)$ and $Bi_m(X,t)$ belong to the Hilbert space of square integrable functions in the domain $0 < X < r_a$ and $0 < t < t_f$:

$$\nabla S[Bi_m(X,t)] = \{a\ l_1(X,1,t)(1-e)KoLu[f(X,1,t)-1] + Lu\, l_2(X,1,t)[1-f(X,1,t)g]\} \qquad (10.a)$$

$$\nabla S[Bi_q\,(X\,,t\,)] = \{a\ \ l_1(X\,,1,t\,)[1-q\,(X\,,1,t\,)] + Lu\ \ l_2\,(X\,,1,t\,)\,Pn[1-q\,(X\,,1,t\,)]\} \qquad (10.b)$$

In this paper, the *Discrepancy Principle* [3-5] is used to specify the tolerance for the stopping criterion of the iterative procedure of the conjugate gradient method. In the Discrepancy Principle, the solution is assumed to be sufficiently accurate when the difference between measured and estimated quantities is of the order of magnitude of the measurement errors.

RESULTS AND DISCUSSIONS

For the results presented below, we examined test-cases involving the drying of a capillary-porous body with dimensions h=0.05m and L=0.5m, made of ceramics, with properties [7]: k=0.34*W/mK*, k_m,=2.4x10^{-7}*kg/ms*^{o}*M*, c=607*J/kgK* r=2.5x10^6*J/kg*, T_0=24^o*C*, u_0=80^o*M*, d = 0.56^o*M/K* and $e = 0.8$. The air conditions were taken as T_s=30^o*C* and u^*=40^o*M* and the applied heat flux as q = 40 *W/m*^{2}. Therefore, the dimensionless numbers appearing in the formulation were Lu=0.2, Pn=0.084, Ko=49 and Q=0.9. The final time is taken as 1785 s, so that the dimensionless final time is t_f = 0.2.

For the estimation of the unknown heat and mass transfer coefficients, we made use of simulated temperature and moisture content measurements. The simulated measurements contained additive, uncorrelated, and normally distributed errors with standard deviation of 1%

of the maximum value of the measured quantity. Hypothetical functions containing discontinuities, which are the most difficult to be recovered through the solution of the inverse problem, were used to generate the simulated measurements. For each sensor, one measurement is considered to be available every Dt = 0.001, which corresponds to a frequency of 0.11 Hz.

Let us consider initially in the analysis the estimation of the heat transfer coefficient $Bi_q(X,t)$, by assuming that the mass transfer coefficient $Bi_m(X,t)$ is exactly known. For this case, $Bi_m(X,t)$ was taken as the same function as for $Bi_q(X,t)$. Figure 1 presents the spatial variation for $Bi_q(X,t)$ at selected times, obtained with temperature measurements (w_q=1 and w_f=0 in equation (3)) of 13 sensors evenly spaced along the body length, at Y = 0.85 (which corresponds to 7.5 mm below the top surface). We note in figure 1 that the estimated function deviates from the exact one near t = 0.2, because of the null gradient at the final time (see equations (9.i,j) and (10.a,b)). On the other hand, quite accurate estimates can be obtained for $Bi_q(X,t)$ by using temperature measurements for other times.

We now consider the estimation of $Bi_m(X,t)$ by using temperature measurements (w_q=1 and w_f=0 in equation (3)), and assuming $Bi_q(X,t)$ as exactly known for the inverse analysis. For this case, $Bi_q(X,t)$ was taken as the same function as for $Bi_m(X,t)$. Figure 2 presents the estimated function obtained with measurements of 13 sensors equally spaced along the body length, at the position Y = 0.85. Figure 2 shows the interesting fact that temperature measurements provide useful information for the estimation of $Bi_m(X,t)$. This is an important result, because quite involved and inaccurate techniques for the measurement of moisture content can be avoided, in favor of inexpensive and accurate temperature measurements, for the estimation of $Bi_m(X,t)$, if $Bi_q(X,t)$ is known. The use of moisture content measurements can result on accurate estimations for $Bi_m(X)$, as illustrated in figure 3; but not for the estimation of $Bi_q(X)$. The estimated function shown in figure 3 was obtained with moisture content measured data (w_q=0 and w_f=1 in equation (3)), of 13 sensors equally spaced along the body length, at the position Y = 0.85.

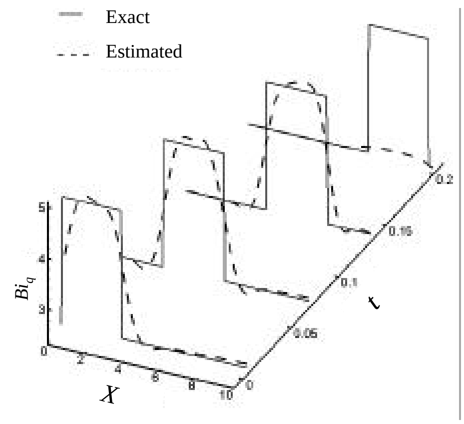

Figure 1. Results obtained for $Bi_q(X,t)$ with known $Bi_m(X,t)$ by using temperature measurements

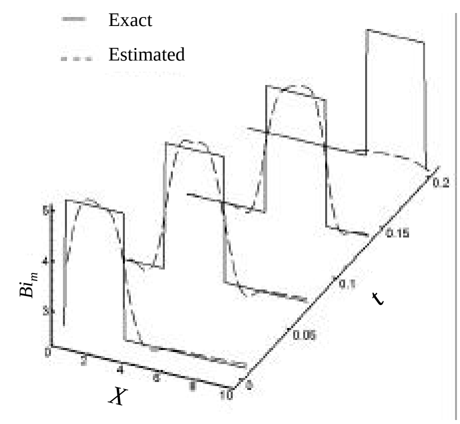

Figure 2. Results obtained for $Bi_m(X,t)$ with known $Bi_q(X,t)$ by using temperature measurements

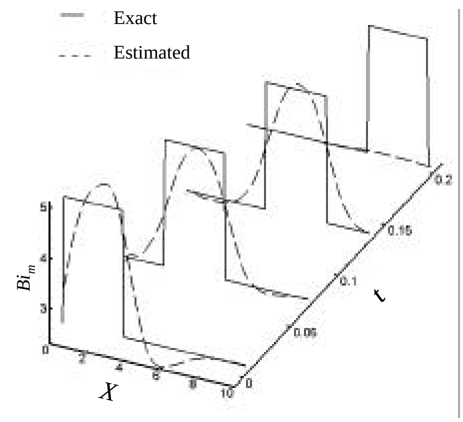

Figure 3. Results obtained for $Bi_m(X,t)$ with known $Bi_q(X,t)$ by using moisture content measurements

The simultaneous estimation of $Bi_q(X,t)$ and $Bi_m(X,t)$ is now examined. For this case, the use of only temperature measurements or only moisture content measurements did not result on accurate estimated functions. Therefore, temperature, as well as moisture content measurements, were required for the simultaneous estimation of $Bi_q(X,t)$ and $Bi_m(X,t)$. Figures 4.a,b present the results obtained for $Bi_q(X,t)$ and $Bi_m(X,t)$, respectively, by using in the inverse analysis simulated measurements of 13 temperature sensors and 13 moisture content sensors. In this case, we used $w_q = 1/M_{max}^2$ and $w_f = 1/C_{max}^2$ in equation (3), where M_{max} and C_{max} are the maximum measured values of temperature and moisture content, respectively. The temperature sensors and the moisture content sensors were located at Y=0.85, evenly spaced along the length of the body. Figures 4.a,b show that quite accurate results can be obtained for the simultaneous estimation of $Bi_q(X,t)$ and $Bi_m(X,t)$, if both temperature and moisture content measurements are used in the inverse analysis. We note however, when we compare figures 4.a,b with figures 1-3, that more accurate estimations can be obtained in cases involving one single unknown function.

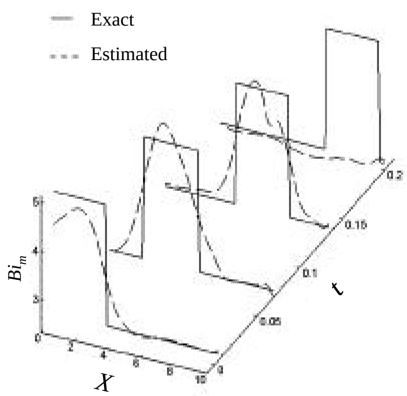

Figure 4.a. Results obtained for $Bi_m(X,t)$ by using temperature and moisture content measurements - simultaneous estimation of $Bi_m(X,t)$ and $Bi_q(X,t)$

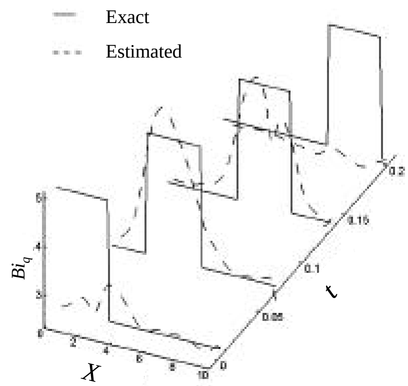

Figure 4.b. Results obtained for $Bi_q(X,t)$ by using temperature and moisture content measurements - simultaneous estimation of $Bi_m(X,t)$ and $Bi_q(X,t)$

CONCLUSIONS

In this paper we solved the inverse problem of simultaneously estimating the mass and heat transfer coefficient at the surface of a drying-capillary porous body. The unknown quantities are treated as functions of time and of the position along the surface open to the surrounding environment. The present inverse problem is solved with the conjugate gradient method of function estimation with adjoint problem.

Results obtained with simulated measurements indicate that the present approach is capable of recovering the heat transfer coefficient, with the use of only temperature measurements, if the mass transfer coefficient is known for the analysis. If the heat transfer coefficient is regarded as known for the inverse analysis, the mass transfer coefficient can be estimated by using either temperature measurements or moisture content measurements. Both temperature and moisture content measurements are required for the simultaneous estimation of the mass and heat transfer coefficients. The present solution approach is stable with respect to measurement errors. Quite accurate results were obtained even for functional forms containing discontinuities, which are the most difficult to be recovered by inverse analysis.

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